



$$f(x) = f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 + \dots + \frac{1}{n!}f^{(n)}(0)x^n + \dots$$

$$= \sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(0)x^k$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

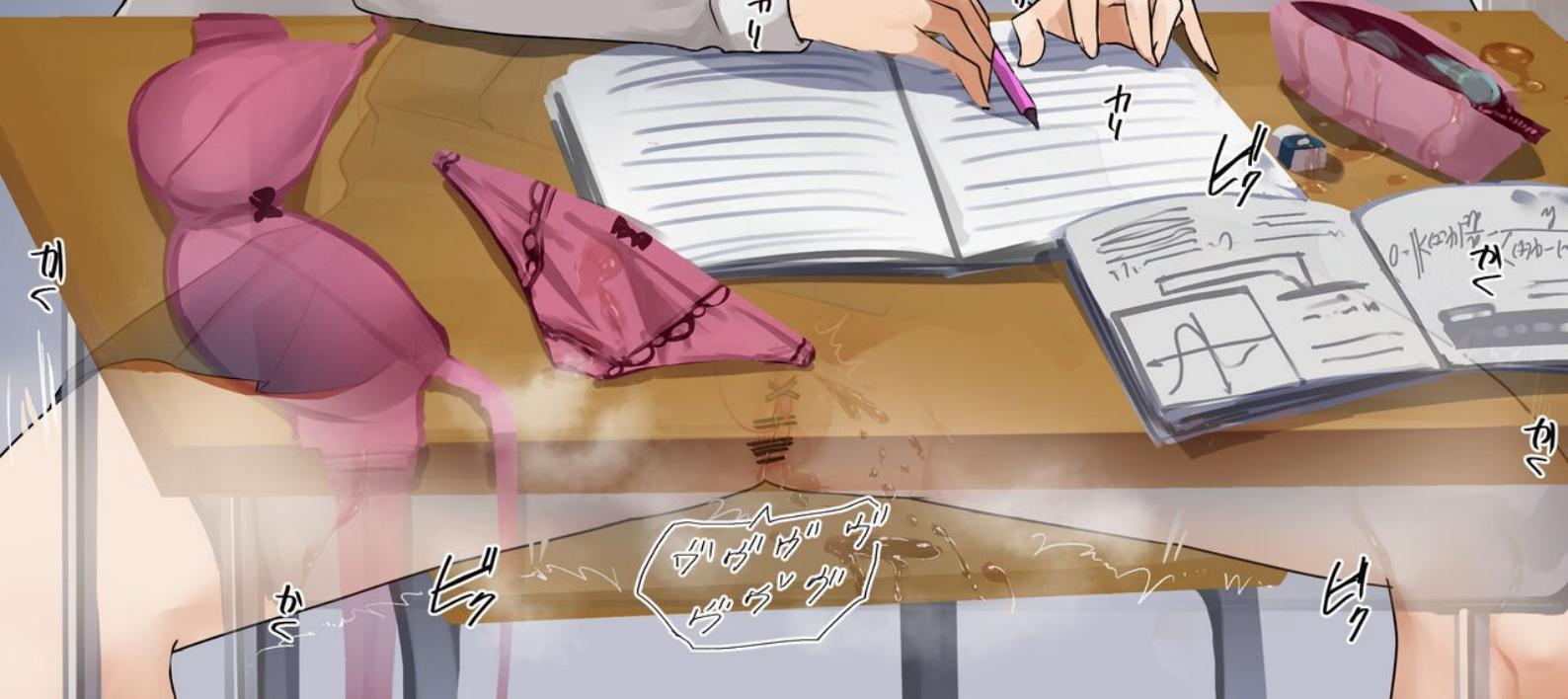
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\therefore e^{ix} = 1 + ix - \frac{x^2}{2!} + \frac{ix^3}{3!} + \frac{x^4}{4!} - \frac{ix^5}{5!} + \dots$$

$$= (1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots) + i(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots)$$

$$= \cos x + i \sin x$$



$$f(x) = f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 + \dots + \frac{1}{n!}f^{(n)}(0)x^n + \dots$$

$$= \sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(0)x^k$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$= \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\therefore e^{ix} = 1 + ix - \frac{x^2}{2!} + \frac{ix^3}{3!} + \frac{x^4}{4!} - \frac{ix^5}{5!} + \dots$$

$$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right)$$

$$= \cos x + i \sin x$$



